Probability: Expected Value
II.C Student Activity Sheet 9: Binomial Probability and Expected Value

At the National Baseball Batting Contest, the organizers have set up game booths for the contestants. Richard wants to win a large stuffed animal. The rules of the game are as follows:

- You are pitched 5 fastballs, and you must hit them into a fair zone to count.
- If you successfully hit all 5 pitches, you win a large stuffed animal.
- If you successfully hit 3 or 4 pitches, you win a small stuffed animal.
- If you successfully hit 1 or 2 pitches, you win a bat-shaped pencil.
- If you miss all the pitches, you do not win a prize.
- The game costs $3 to play (each set of 5 fastballs).

1. What are the possible outcomes that Richard can have on his 5 swings? Explain or show how you arrived at your answer?

2. What is the probability of Richard successfully hitting the following? Justify your answers.

   a. All 5 pitches?

   b. 4 pitches?

   c. 3 pitches?

   d. 2 pitches?

   e. 1 pitch?

   f. 0 pitches?

3. Suppose Richard tries the game 10 times. How many times do you expect him to successfully hit the given amounts of times below? Justify your answers.

   a. 5

   b. 4

   c. 3

   d. 2

   e. 1

   f. 0

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4. Everyone has the same chance as Richard to win. If 160 people each play the game once, how many large stuffed animals will be won? Explain your reasoning.

5. If 160 people each play the game once, how many small stuffed animals will be won? Explain your reasoning.

6. If 160 people each play the game once, how many bat-shaped pencils will be won? Explain your reasoning.

7. If a large stuffed animal costs the game organizers $6, a small stuffed animal costs $1, and a pencil costs $0.25, how much profit do you expect the group to make from the 160 players?

8. **EXTENSION:** For the game described in which you are pitched 5 fastballs, you found the probabilities of successfully hitting 5, 4, 3, 2, 1, or 0 pitches. Suppose the game is changed and you are pitched various number of pitches. Find the probabilities of each outcome and look for a pattern of numbers. (Because you are looking for a pattern, do not simplify the fractions.)
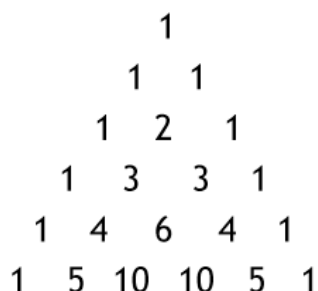
1 pitch:

2 pitches:

3 pitches:

4 pitches:

Consider the geometric arrangements known as Pascal's triangle below. Decide how the values in the triangle are generated. Then compare the values you found for the probabilities above to the values in the triangle. What do you notice?

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 \\
\end{array}
$$

If you had to successfully hit 6 of 6 pitches to win the large stuffed animal, what is the probability of winning the large stuffed animal? Use Pascal's triangle in your solution.

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9. **EXTENSION:** Research the use of Pascal's triangle. Prepare a short class presentation regarding your findings. Be sure to include the connection to combinatorics.

10. **EXTENSION:** Investigate the probabilities and associated expected values related to winning a state or local lottery game. Prepare a short class presentation about the probable outcomes of event of winning the lottery.

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